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# Consumption Paths under Prospect Utility in an Optimal Growth Model\*

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## Abstract

This paper studies the Cass-Koopmans-Ramsey model of optimal economic growth in the presence of loss aversion and habit formation. The representative agent's preferences for consumption can be gradually varied between the standard constant intertemporal elasticity of substitution (CIES) case and Kahneman and Tversky's prospect utility. We find that the transitional dynamics of optimal consumption paths differ distinctly from the standard model, in particular consumption smoothing is more pronounced. We also show that prospect utility can cause the economy to remain in a steady state with low consumption and low capital.

*JEL-Classification:* E21; O41.

*Keywords:* Ramsey growth model; prospect theory; loss aversion; optimal consumption.

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# 1 Introduction

Reference points and the different perceptions of gains and losses play an important role in actual decision-making, see, e.g., Kahneman and Tversky (2000). As K. E. Boulding (1981, p. 108) (one of the founders of modern evolutionary economics and former president of the American Economic Association) puts it: “...the perception of potential threats to survival may be much more important in determining behavior than the perceptions of potential profits, so that profit maximization is not really the driving force. It is fear of loss rather than hope of gain that limits our behavior.” These characteristics are often ignored in standard dynamic macroeconomics which do not take into account the role of positional concerns in consumption. This creates two main issues: First, the empirical evidence suggests the relationship between growth and happiness is not straightforward. When higher income growth drives growth in material aspirations, growth does not need to increase happiness (see, e.g., Easterlin, 2001). This makes the welfare effects of economic growth policies difficult to interpret. Second, when all economic agents’ preferences exhibit positional concerns, the dynamic consumption decision changes and, therefore, the growth process will be affected.

In this paper, we want to address these two problems. We study optimal consumption paths in a growth model with a representative agent to determine the macroeconomic effects if all agents exhibit loss aversion. To this end, we consider preferences that can be varied gradually between the two polar cases of (a) standard time-separable preferences with CIES instantaneous utility and (b) the experimentally validated prospect utility function of Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Kahneman and Tversky’s prospect theory builds *inter alia* on the evidence that economic agents value their prospects in gains and losses relative to a reference point and that losses loom larger than gains.

The main findings of our paper are as follows. First, we show that a neo-classical growth model with a general utility function and past consumption as a reference point exhibits a unique optimal solution. Second, an economy with loss-averse agents might remain in a low capital–low consumption steady state. The reason is that very loss-averse consumers are reluctant to reduce their consumption today to achieve a higher steady state. Third, the transitional dynamics of our model differ substantially not only from the standard model but also from an economy with habit formation, the most common specification of utility that is *nonseparable* across time. In particu-

lar, the presence of loss aversion leads to strong consumption smoothing, as consumers avoid reductions in consumption. We find that economies with relatively high income smooth consumption to a larger extent than economies with relatively low income, which is consistent with microeconomic evidence in Gervais and Klein (2010). Furthermore, the consumption reaction is more extreme in downturns which matches the stylized fact that recessions are more pronounced than booms over the business cycle.

There is an increasing interest in (growth) models in which the agents' utility functions are non-standard and current utility also depends on past consumption or, more generally, where positional concerns play a role in the consumption-saving choice. Easterlin (1995), Clark and Oswald (1996) and Layard (2003) provide evidence, both from economics and psychology, that positional concerns are important in understanding actual consumption patterns. In particular, psychological research into happiness, see, e.g., Easterlin (1974), Frank (1997) and Kahneman and Tversky (2000), provide evidence that utility is reference-based (both internally through habits and externally through comparison). Theoretical approaches are, for instance, Ryder and Heal (1973) and Boyer (1978) allowing for habit formation, Laibson (1997) and Barro (1999) studying hyperbolic discounting, Koopmans (1960), Uzawa (1968) and, more recently, Mausumi (2003) dealing with recursive preferences and marginal impatience, Shi and Epstein (1993) incorporating recursive preferences and habit formation, and De la Croix and Michel (1999) investigating optimal growth under hereditary tastes. Closely related to our approach are the papers by Carroll et al. (2000) and Alvarez-Cuadrado et al. (2004) who study the implications of habit formation or 'Keeping up with the Joneses' in a growth context; though none of these papers explores the prospect utility point of view in an economic growth context.

Although prospect theory is not yet common in the context of macroeconomic growth models, there are many fields where it (and, in particular, loss aversion) has been successfully applied, e.g., asset pricing (Benartzi and Thaler, 1995, Barberis et al., 2001), tax evasion (Yaniv, 1999) or monetary policy (Surico, 2007). Camerer (2004) gives an overview of many more phenomena which are inconsistent with expected utility theory but can be explained by 'thinking in differences' and, in particular, loss aversion, e.g., asymmetric price elasticities in consumer goods, insensitivity of consumption paths to bad income news, downward-sloping labor supply curves, the disposition effect, status quo bias, and default bias in decision-making.

The remainder of this paper is organized in the following fashion. Sec-

tion 2 introduces the model along with a convenient reformulation that allows the application of value function techniques. Section 3 derives qualitative properties of optimal consumption paths under prospect utility. Section 4 presents a numerical example. Section 5 concludes and discusses extensions of our model. All proofs are collected in the Appendix.

## 2 The model

Consider a version of the Ramsey optimal growth model in which the representative agent's preferences for consumption are represented by a utility function that depends on current as well as last period's consumption.<sup>1</sup>

Capital stock and consumption in period  $t$  are denoted  $k_t$  and  $c_t$ . The instantaneous utility function is denoted  $U(\cdot, \cdot) : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$ .  $U$  is assumed to be a continuous function that is strictly decreasing in the first component and strictly increasing in the second component. An increase in  $c_{t-1}$ , which provides the reference level of consumption, decreases  $U(c_{t-1}, c_t)$  for fixed  $c_t$ . But for any given  $c_{t-1}$ , an increase in current consumption  $c_t$  will yield higher utility. The production function  $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is increasing and continuous with  $f(0) = 0$  (measuring  $f(\cdot)$  net of depreciation). We further assume that there is some maximum sustainable capital stock  $\bar{k} > 0$ , i.e.,  $f(\bar{k}) = \bar{k}$  and  $f(k) \leq k$  for all  $k \geq \bar{k}$ .

The economic agent aims to maximize the present value of utility from consumption, discounted at rate  $0 < \beta < 1$ , for a given past (reference) consumption  $c_{-1}$  and a current capital stock  $k_0 \geq 0$ :

$$\begin{aligned} \sup_{\{c_t, k_{t+1}\}_{t \geq 0}} \quad & \sum_{t=0}^{\infty} \beta^t U(c_{t-1}, c_t) \\ \text{s.t.} \quad & c_t + k_{t+1} \leq f(k_t), \\ & \text{and } c_t \geq 0, k_{t+1} \geq 0 \text{ for all } t = 0, 1, \dots \end{aligned} \tag{1}$$

The budget constraint  $c_t + k_{t+1} \leq f(k_t)$  is assumed to be binding. If it

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<sup>1</sup>Prospect theory provides only little guidance on how the reference point is determined (see the discussion in Rosenblatt-Wisch, 2008, p. 1144). In our dynamic framework, we assume that the status quo realized in every period serves as the new reference point. Fuhrer (2000) presents further supporting evidence: Using an empirical model with habit formation, he cannot reject the hypothesis that habit is completely pinned down by the consumption in the previous period.

were not binding, increasing the investment  $k_{t+1}$  would strictly increase the set of feasible plans after the current date without decreasing current utility.

This model can be solved as follows, provided there is a capital stock  $k_{-1}$  such that  $c_{-1} + k_0 = f(k_{-1})$ . (If this equation has a solution, it is unique by our above assumptions on the production function.)

We define  $x_t = (x_t^1, x_t^2)$  and let  $F(x_t, x_{t+1}) := U(f(x_t^1) - x_t^2, f(x_{t+1}^1) - x_{t+1}^2)$ . Note that  $x_t^1$  stands for  $k_{t-1}$  and  $x_t^2$  for  $k_t$ . Given  $x_0$ , the representative agent maximizes

$$\begin{aligned} \sup_{\{x_{t+1}\}_{t \geq 0}} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}) \\ \text{s.t. } x_{t+1} \in \Gamma(x_t) \text{ for all } t = 0, 1, \dots \end{aligned} \quad (2)$$

where

$$F(x_t, x_{t+1}) = U(f(x_t^1) - x_t^2, f(x_{t+1}^1) - x_{t+1}^2) \quad (3)$$

and

$$\Gamma(x) = \{z = (z^1, z^2) \in \mathbb{R}^2 \mid 0 \leq z^1 \leq x^2, 0 \leq z^2 \leq f(z^1)\}. \quad (4)$$

The set  $\Gamma$  is increasing because free disposal of capital is allowed. The second intertemporal maximization problem appears to be more general than the first. However, the preceding discussion makes clear that one can always assume  $x_{t+1}^1 = x_t^2$  along an optimal path. In our numerical study (Section 4) this property is used to improve computational efficiency.

For representation (2), the Bellman equation takes on the familiar form,

$$V(x_t) = \sup_{x_{t+1} \in \Gamma(x_t)} [F(x_t, x_{t+1}) + \beta V(x_{t+1})] \quad (5)$$

where, however, the value function  $V$  depends on a two-dimensional variable.

Existence and uniqueness of a function  $V^*$  solving (5) can be proved as usual by Blackwell's sufficient conditions for a contraction: Define the operator

$$(TV)(x) = \sup_{z \in \Gamma(x)} [F(x, z) + \beta V(z)]. \quad (6)$$

$T$  is monotone ( $V \leq W$  implies  $TV \leq TW$ ), and has the discounting property ( $T(V + a)(x) \leq (TV)(x) + \beta a$ ) because  $0 < \beta < 1$ .

Define  $X(k) = [0, k] \times [0, k] \subset \mathbb{R}_+^2$  for every  $k \geq \bar{k}$ . Then, by our definition of the production function  $f$ ,  $\Gamma(x) \subset X(k)$  for all  $x \in X(k)$ . Thus  $X(k)$  is a state space of the model for every sufficiently large  $k$ . Since  $U$  is continuous,

it is bounded on  $X(k)$ . Therefore  $T$  maps the space of bounded real-valued functions on  $X(k)$  into itself. This ensures existence and uniqueness of a bounded solution  $V^*$  to (5). Monotonicity properties of  $V^*$  can be obtained in the usual way. Denote by  $H_1$  the complete space (with respect to the sup-norm) of functions  $V : X(k) \rightarrow \mathbb{R}$  which are non-increasing in the first and non-decreasing in the second argument. Further let  $H_2 \subset H_1$  denote the space of functions where the monotonicity is strict in each component. The assumed properties of  $U$ ,  $F$  and  $\Gamma$  ensure that  $T(H_1) \subset H_2$ . This implies  $V^* \in H_2$ . Continuity of  $V^*$  can be proved along the lines of Stokey, Lucas and Prescott (Theorem 4.6, 1989) because  $\Gamma(x)$  is compact and the correspondence  $x \mapsto \Gamma(x)$  is continuous.

To summarize the discussion, we state:

**Proposition 1.** *The optimization problem (5) with state space  $X(k) = [0, k] \times [0, k]$ , where  $k \geq \bar{k}$ , has a unique bounded solution  $V^*$ . The solution is continuous, strictly decreasing in the first argument and strictly increasing in the second argument.*

Stokey, Lucas and Prescott (Theorem 4.6, 1989) ensures that the set of optimal decisions  $G : X(k) \rightarrow X(k)$ ,  $k \geq \bar{k}$ , defined by

$$G(x) = \{z \in \Gamma(x) \mid V^*(x) = F(x, z) + \beta V^*(z)\},$$

is nonempty, compact-valued and upper hemi-continuous. This finding guarantees that a solution to the problem (2) exists. A unique policy function can be extracted by imposing the additional assumption that the economic agent always chooses the highest consumption if more than one choice gives the same value. By continuity and compactness such a consumption choice is feasible and unique.

### 3 A specific loss-aversion utility

We describe the qualitative properties of optimal consumption and capital paths when agents are loss averse for a specific utility function which has two main features. First, it combines the commonly used CIES utility function with preferences akin to Kahneman and Tversky's prospect theory (1979/1992). Second, it allows to scale the degree of prospect utility such that the model can be gradually varied between the two polar cases.

The instantaneous utility function

$$U(c_{t-1}, c_t) = (1 - \alpha)u(c_t) + \alpha v(c_t - c_{t-1}) \quad (7)$$

weighs instantaneous utility  $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$  against loss aversion with reference level given by previous period's consumption

$$v(\Delta c_t) = \begin{cases} \Delta c_t & \text{if } \Delta c_t \geq 0, \\ \phi \Delta c_t & \text{if } \Delta c_t < 0, \end{cases} \quad (8)$$

where  $\Delta c_t := c_t - c_{t-1}$  and where the parameter  $\alpha$  is between zero (no prospect utility) and one (pure prospect utility maximizer). This modification allows at the same time for loss aversion and decreasing marginal utility. The second component  $v(\Delta c_t)$  of the utility function is a piecewise-linear approximation of Kahneman and Tversky's kinked power utility function which weighs negative differences in consumption heavier than gains where the loss aversion parameter  $\phi > 1$ . The core of prospect theory for our purposes is the notion of loss aversion and thinking in differences, i.e., the existence of a reference point. Hence, to capture the asymmetry between gains and losses, this simple piecewise-linear function entails the essential argument of loss aversion (see Rosenblatt-Wisch, 2008, p. 1143).

### 3.1 Euler equation and steady state

We derive the first order conditions for the instantaneous utility function (7) where the capital accumulation constraint is binding. Along an optimal consumption path, where  $\Delta c_t, \Delta c_{t+1}$ , and  $\Delta c_{t+2}$  are different from zero, the Euler equation is given by

$$\begin{aligned} u'(c_t) - \beta f'(k_{t+1})u'(c_{t+1}) \\ = \frac{\alpha}{1-\alpha} [-v'(\Delta c_t) + \beta v'(\Delta c_{t+1}) + \beta f'(k_{t+1}) (v'(\Delta c_{t+1}) - \beta v'(\Delta c_{t+2}))]. \end{aligned} \quad (9)$$

This Euler equation differs from the standard formulation in a Ramsey model. Consumption is no longer time-separable since the objective function is now dependent not only on  $c_t$  and  $c_{t+1}$  but also on  $c_{t+2}$  and  $c_{t-1}$ . Previous decisions about consumption and capital change the reference point which affects current and future expected utility. If a consumer considers a reduction in consumption today versus an increase in consumption tomorrow, he takes



into account that this implies a utility loss  $u'(c_t)$  today and a utility gain  $\beta f'(k_{t+1})u'(c_{t+1})$  tomorrow, like in the standard model. The right hand side of (9) captures the prospect elements, their relative weight is  $\alpha/(1-\alpha)$  in (7). By reducing today's consumption, the economic agent suffers from an additional utility loss  $-v'(\Delta c_t)$ . However, the reference point is lower for tomorrow's consumption which yields a gain of  $\beta v'(\Delta c_{t+1})$ . Instead, the increase in tomorrow's consumption will increase the reference point tomorrow which reduces utility the day after tomorrow by  $\beta f'(k_{t+1})(v'(\Delta c_{t+1}) - \beta v'(\Delta c_{t+2}))$ .

The Euler equation (9) may not hold with equality in time periods where in the optimum  $c_t = c_{t+1}$ . The reason is that  $v(\Delta c_t)$  is not differentiable at  $\Delta c_t = 0$  because of the presence of loss aversion. Recall that the derivative of  $v(\Delta c_t)$  is equal to  $\phi > 1$  (resp. 1) when zero is approached from the left (resp. right). A consumer will find it optimal to choose  $\Delta c_t = \Delta c_{t+1} = \Delta c_{t+2} = 0$  if the following inequality holds,

$$\begin{aligned} & \beta - \phi + \beta f'(k_{t+1})(1 - \beta\phi) \\ & \leq \frac{1-\alpha}{\alpha} [1 - \beta f'(k_{t+1})] u'(c) \leq \beta\phi - 1 + \beta f'(k_{t+1})(\phi - \beta), \end{aligned} \tag{10}$$

where  $c_t = c_{t+1} = c$ . To derive equation (10), start out from a situation where  $c_t = c_{t+1} = c$ . Then neither a small reduction nor a small increase in  $c_t$  can increase utility in the optimum. Since the loss aversion parameter  $\phi > 1$ , it is straightforward to see that a positive range of parameter values must exist such that (10) holds. In particular, we observe that a consumer may choose (temporarily) a constant consumption profile in the transition process where  $\beta f'(k_{t+1}) \neq 1$ . The reason is the presence of loss aversion: The kink in the utility function (for  $\phi > 1$ ) makes a consumer reluctant to follow a changing consumption path even if the marginal product of capital would induce them to so in the standard model, i.e., when  $\beta f'(k_{t+1})$  is different from one. Such a 'plateau building' in consumption may occur when the economy-wide capital stock is above as well as when it is below the steady state. In the absence of loss aversion, this feature may not be present. If  $\phi$  approaches one, which corresponds to the case with habit formation, the left- and the right-hand side of (10) coincide. Then consumption will only be constant if the economy is in the steady state.

Interestingly, when the loss aversion parameter  $\phi$  is sufficiently high, the aforementioned effects are so strong that the consumption path stays not only temporary but permanently on a plateau different from a neoclassical steady state. In that case, the economy stays in a 'poverty trap.' If the

economy starts at a capital stock level below the steady state and  $k_0 = k_1$ , it would be necessary to reduce consumption tomorrow to reach the steady state determined by  $\beta f'(k^*) = 1$ . If loss aversion is very high, however, individuals are more concerned about the initial consumption drop than about the future gain in consumption. Note that this result does not depend on the discount rate  $\beta$ : Without loss aversion, the individual would value the future consumption gains always more when starting at  $k_0 < k^*$ . The reluctance to suffer an initial loss results in the economy remaining in a low capital/low consumption state.<sup>2</sup> In contrast, if  $\phi$  is sufficiently low, a unique steady state exists.

**Proposition 2.** *An economy with loss aversion given by utility function (7) has a unique steady state with capital stock  $k^*$  given that  $\beta\phi \leq \frac{1-\alpha}{\alpha}u'(c^*) + 1$ , where  $c^* = f(k^*) - k^*$  and  $\beta f'(k^*) = 1$ .*

**Proof.** *See Appendix.*

Under the sufficient assumption  $\beta\phi \leq \frac{1-\alpha}{\alpha}u'(c^*) + 1$ , an economy where individuals show loss aversion converges to the same steady state as in the neoclassical growth model.

## 4 Numerical simulations

To better understand the quantitative macroeconomic implications of loss aversion, we carry out a numerical simulation study. The results obtained in the preceding sections ensure existence of a solution to the optimization problem which can be characterized by the value function. All simulations are based on an approximation of the value function through iteration of the Bellman operator on a grid of  $1,000 \times 1,000$  equidistant points on the set  $[0, 1] \times [0, 1]$ . This simple method turns out to be sufficient here.<sup>3</sup> The parameters chosen are standard: We set  $\beta = 0.95$ ,  $\sigma = 0.5$  and the production function to  $f(k) = \sqrt{k}$ . In accordance with Tversky and Kahneman (1992), we set the loss aversion parameter  $\phi = 2.25$ . As a sensitivity analysis we ran

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<sup>2</sup>Numerically, consumption remains constant below the steady state in the following example. Set  $f(k) = \sqrt{k}$ ,  $\beta = 0.95$ ,  $\sigma = 0.5$ ,  $\alpha = 0.9$ , and  $\phi = 10$ . Let  $k_0 = k_1$  such that  $c_0 = \sqrt{k_0} - k_0$ . Our simulations show that for  $k_0 = k_1 > 0.811k^*$  consumption stays constant at  $c = c_0$ .

<sup>3</sup>The software is available at [www.schenk-hoppe.net/software.html](http://www.schenk-hoppe.net/software.html).

simulations for higher values of  $\sigma$  as well and found that there are no qualitative change. In line with our expectations, consumption follows a much smoother path, and the speed of convergence is lower for all specifications.

Each figure contains three paths, one for each of the three different initial values of the capital stock  $k_0$  and consumption  $c_0$ . The dashed line  $(k_0, c_0) = (0.005, 0.01)$  corresponds to the case of a growing economy because both  $k_0$  and  $c_0$  are below their steady state values  $(k^*, c^*) = (\beta^2/4, \beta/2 - \beta^2/4) \approx (0.226, 0.249)$ . The solid line  $(k_0, c_0) = (0.3, 0.05)$  represents the case where the reference point  $c_0$  is low and the inherited capital stock  $k_0$  is high. Consequently,  $k_1$  and  $c_1$  are larger than their corresponding steady state values and the economy follows a declining path towards the steady state. Finally, the dotted line corresponding to  $(k_0, c_0) = (0.3, 0.45)$  highlights a situation in which the reference point  $c_0$  is above and  $k_1 = 0.097$  is below the steady state. These values imply that  $c_1$  must lie below the steady state and, therefore, will be lower than  $c_0$ . (Otherwise, the economy could not converge on an increasing path towards the steady state.) This case can be interpreted as an unanticipated drop in capital.

Figure 1 depicts the optimal consumption paths for a representative agent with loss aversion. The weight of the prospect utility part is  $\alpha = 0.9$ . Figure 2 shows the consumption choice of an individual with habit formation. The prospect utility function (7) contains habit formation as a special case if  $\phi = 1$ . The utility function depends on the habit stock (the consumption level of the previous period). However, in absolute terms and in contrast to loss aversion, there is no asymmetry between negative and positive changes in consumption. Figure 3 shows the dynamics of the standard neoclassical growth model as a benchmark. This reference case (with  $\alpha = 0$ ) coincides with the saddle path of the standard model from period  $t = 1$  on.

Even in this simple deterministic setting we observe that prospect theory has a drastic impact on the transition path, Figure 1. The reference point and the asymmetry between losses and gains play a crucial role: When the reference point is low and the inherited capital stock is high (solid line), the initial rise in consumption is much smaller than in the Ramsey case as consumption will eventually be lower in the steady state. Since the agent is loss averse, he wants to reduce the amount of negative consumption changes in the future, which implies that  $c_1$  is lower than in the Ramsey case. In other words, the presence of loss aversion decreases the intertemporal elasticity of substitution and the speed of convergence. There is a second important difference to the standard model: the transition path differs in its shape.

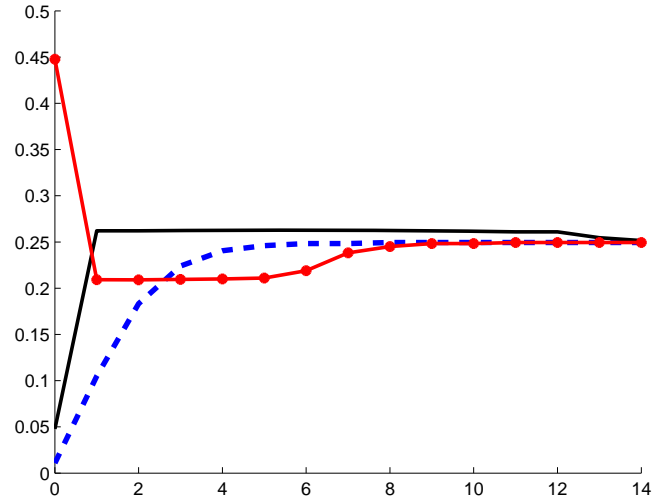


Figure 1: Loss aversion case ( $\alpha = 0.9$  and  $\phi = 2.25$ ). Optimal consumption paths for different pairs of initial capital stock and consumption  $(k_0, c_0)$ : dashed line  $(0.005, 0.01)$ , solid line  $(0.3, 0.05)$ , dotted line  $(0.3, 0.45)$ .

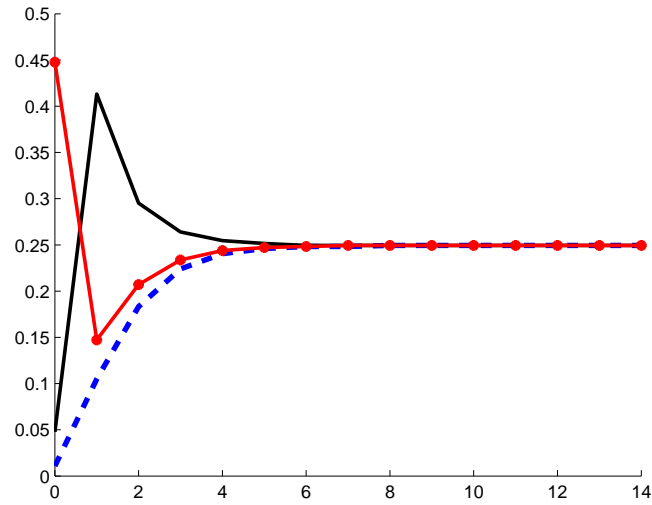


Figure 2: Habit case ( $\alpha = 0.9$  and  $\phi = 1$ ). Optimal consumption paths (initial values as in Figure 1).

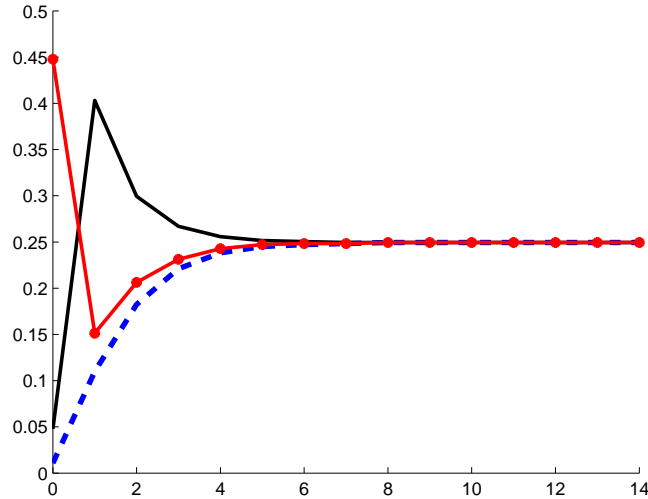


Figure 3: Standard case ( $\alpha = 0.0$ ). Optimal consumption paths (initial values as in Figure 1).

The optimal consumption path is chosen in such a way that the reduction in consumption is only realized in the future. Since future losses are discounted, this reduction is less painful from today's perspective than an immediate drop. This entails a 'consumption plateau.' As demonstrated using the Euler condition (10), such a behavior may indeed be optimal along a path (caused by the non-differentiability of the utility function at  $\Delta c = 0$ ). In the opposite case, when the reference point  $c_0$  is high and the inherited capital stock is low (the dotted line in Figure 1), a loss-averse person will not decrease consumption in  $t = 1$  as much as a Ramsey consumer does. Since reducing consumption hurts more than twice as much as gaining the same amount, an optimal path requires that consumption does not decrease as much as in the standard case and stays constant thereafter to build up capital.

We observe again a consumption plateau as long as enough capital is built up and the economy converges to a steady state. However, a comparison of the solid to the dotted line shows that the consumption reaction is more extreme in a downturn. When both the reference point and consumption are low (dashed line), the transition path of a loss-averse consumer is close to the standard model. Intuitively, when consumption exhibits steady growth and there are no shocks, the loss aversion component of the utility function

(7) does not play a role. In all three cases (and in accordance with Prop. 2), the economy eventually settles on the same steady state as in the standard Ramsey model (poverty traps cannot occur for these parameter values).

Comparing the optimal choice of a representation agent with habit formation (Figure 2) and a standard Ramsey consumer (Figure 3), we see that the transitional dynamics in both cases are quantitatively very similar.<sup>4</sup> If the reference point is very different from the consumption thereafter, the economic agent wants to smooth consumption in such a way that the speed of convergence is lower than in the Ramsey model. However, consumption smoothing is much less pronounced than in the model with loss aversion where  $\phi = 2.25$ . Solely thinking in differences does not produce dynamics very different to the standard case and, in particular, it does not exhibit excessive consumption smoothing.

In the standard permanent income hypothesis model with rational expectations, consumption jumps immediately in response to current “news” about lifetime resources, a direct implication of the random-walk property of consumption. Although our model assumes rational expectations, consumption is much smoother than income. The Ramsey model, in contrast, has the feature that income is as smooth as consumption. Gervais and Klein (2010) show that households with relatively high income, smooth consumption to a larger extent than households with relatively low income. Applying these findings to our economic growth framework, households with a low capital stock (i.e., with an income below steady state in period 1) should smooth their consumption less than agents with a capital stock above the steady state (relatively rich and high income). Indeed, this asymmetry is reflected in our model: Compare the dashed line with the solid line in Figure 1, the latter exhibits obviously more consumption smoothing. In the standard Ramsey model, instead, households show the same consumption smoothing pattern below or above the Ramsey steady state.

**Unexpected changes to productivity.** To gain further intuition about the implications of loss aversion, we consider a change in productivity which is unanticipated by the representative agent. Let us assume that the production function is given by  $A\sqrt{k}$  with  $A$  a parameter that can vary over time. The

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<sup>4</sup>This result is partly due to the fact that habit or loss elements enter the utility function (7) in an additive way. If we chose a multiplicative formulation (as, e.g., in Carroll et al., 2000 or Alvarez-Cuadrado et al., 2004) the effect of habit would be much stronger. However, this reinforces our results: We are able to find a quantitatively important impact of loss aversion although the additive formulation of the utility function works against it.

previous analysis was concerned with a constant  $A = 1.0$ .

Two scenarios are simulated: At time  $t = 0$ , the economy is in the steady state corresponding to the case  $A = 1$ :  $(k^*, c^*) = (\beta^2/4, \beta/2 - \beta^2/4) \approx (0.226, 0.249)$ . At time  $t = 1$ ,  $A$  increases from 1.0 to 1.5 (Scenario 1) or  $A$  decreases from 1.0 to 0.5 (Scenario 2). Figure 4 depicts the dynamic of the saving rate and consumption.<sup>5</sup>

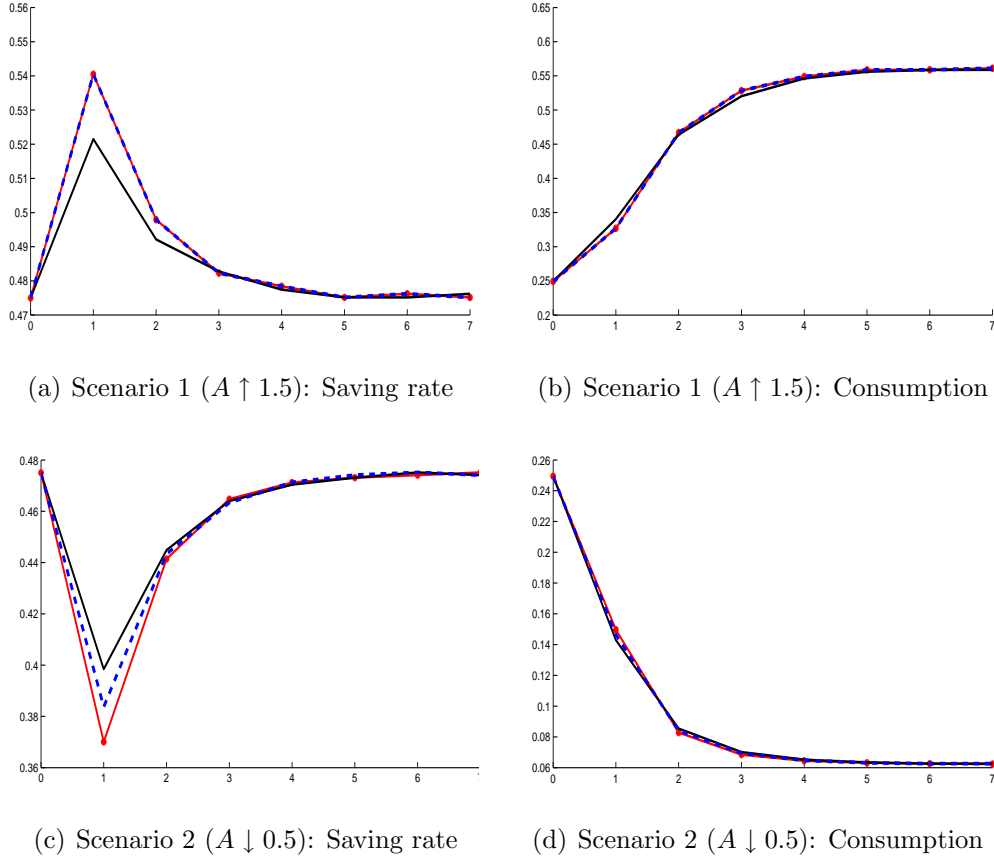


Figure 4: Unanticipated change in productivity at time  $t = 1$ . The capital stock  $k_1$  and the reference level for consumption  $c_0$  are given by the long-run steady state  $(k^*, c^*)$ . Loss aversion (dotted, red), habit formation (dashed, blue), and standard case (solid, black).

<sup>5</sup>Note that in Figure 4 we plot the loss, habit, and standard cases within the same panel. The reference point is the same and pinned down by the steady state for  $A = 1$ .

When productivity increases, the behavior in the habit and loss case coincide, see Figure 4(a) and (b). This is in line with common intuition: As the income shock is positive and the economy is growing to a higher steady state, downside risks and loss aversion have no effect. The increase in the saving rate in both cases is stronger than in the standard case, which confirms the results of Carroll et al. (2000) who show that habits imply a sluggish response of consumption to income shocks, consistent with the stylized facts. The same is true for loss-averse consumers in our model: consumption is smoother because outcomes are valued as differences.

The dynamic is different under a negative technology shock, Figure 4(c) and (d). The loss-averse agent would like to avoid a reduction in consumption. Therefore, after a negative income shock, the cuts in consumption are smoothed and delayed to make them less painful (because the future is discounted). The saving rate is lowered the most in the loss case (because losses loom larger than gains) and consumption is cut by less than in the habit or standard case. Consumption responds in an even more sluggish way than with habit formation (which already exhibits more consumption smoothing than the standard case).

## 5 Discussion and conclusions

The model presented in this paper incorporates prospect utility into the neoclassical growth model. We prove the existence of an optimal consumption policy for general utility functions with past consumption as a reference point, without imposing any assumptions on differentiability. The model can explain the stylized fact of consumption smoothing and, in this respect, is superior to the Ramsey model. Empirical studies have shown that consumption is less volatile than income. In our model, the economic agent's income is generated through the capital stock. Consumption is much smoother than the capital stock, whereas in the Ramsey model consumption and capital stock show almost the same dynamics.

Loss aversion only becomes relevant (in contrast to models with habit formation) when reductions in consumption are possible. In this deterministic model, consumption downturns are only possible when the reference point is high. We expect the same effects to be present, however, in a stochastic version of this model. As the individual wants to avoid reductions in consumption, he will increase savings as a way of self-insurance. Hence, the



behavior of loss-averse consumers is governed by an insurance motive. Our simulation study indicates that the consumption reaction is more extreme in downturns which agrees with the stylized fact that the outset of recessions are more pronounced than that of booms.

The latter finding has important implications for the interpretation of consumption patterns in the light of the Cass-Koopmans-Ramsey model (Ramsey, 1928, Cass, 1965, Koopmans, 1965) with textbook utility function. In the model with an (empirically unjustified) additive-separable utility function, the strong reduction of consumption in a recession may only be explained by assuming the presence of a strongly negative technology shock. Consumption would erroneously be associated with two corresponding steady states and a transition from high to low consumption. These results indicate that stochastic growth—or real business cycle models—with loss-averse agents would require less severe negative technology shocks to explain macroeconomic data.

Our paper suggests several directions for future research, in particular, those focussing on inequality. First, differences in preferences would be worth studying. If only a certain number of agents is loss-averse, we still expect to observe loss-averse behavior on the aggregate level, though at a reduced level. However, ‘poverty traps’ are likely to be ruled out since savings of the Ramsey agents would cause the capital stock to rise. An even more interesting extension would be to analyze endowment inequality where individuals share the same preferences. Heterogeneity in wealth implies that both consumption levels and reference points differ across individuals; this turns out crucial for individual savings and thus gives predictions on the evolution of inequality. Finally, our paper may be seen as a first step towards a more demanding analysis of the stochastic optimal growth model in which loss aversion may provide an insurance motive.

## A Proof of Proposition 2

Assume  $c_t = c_{t+1} = \dots = c_{t+T} = c$  and  $k_{t+1} = k$  with  $k$  given by  $c = f(k) - k$ .

Consider first the case  $k > k^*$  in which  $\beta f'(k) < 1$  because  $\beta f'(k^*) = 1$ . For a stationary consumption path to be optimal, the consumer must not be able to improve his utility by increasing  $c_t, c_{t+1}, \dots, c_{t+T-1}$  and reducing  $c_{t+T}$  the day after tomorrow. As  $c_t = f(k_t) - k_{t+1}$ , the reduction of  $c_{t+T}$  is determined by  $dc_{t+T} = \Pi_{j=1}^T f'(k_{t+j})dc_t + \Pi_{j=2}^T f'(k_{t+j})dc_{t+1} + \dots + f'(k_{t+T})dc_{t+T}$

(holding  $k_{t+T+1}$  constant) where, by definition,  $dc_t = dc_{t+1} = \dots = dc_{T+t-1}$ . We obtain

$$\begin{aligned} & \frac{1-\alpha}{\alpha} \frac{1-\beta^T}{1-\beta} u'(c) + 1 \\ & \leq \frac{1-\alpha}{\alpha} \beta^T \frac{1-f'(k)^{T+1}}{1-f'(k)} u'(c) + \beta^T (\phi - \beta) \frac{1-f'(k)^{T+1}}{1-f'(k)} + \beta^T \phi. \end{aligned}$$

Recall that  $\beta f'(k) < 1$ , and note that  $\beta^T (1 - f'(k)^{T+1})$  can be rewritten as  $\beta^T - [\beta f'(k)]^T f'(k)$ . Letting  $T \rightarrow \infty$ , we find

$$\frac{1-\alpha}{\alpha} \frac{1}{1-\beta} u'(c) + 1 \leq 0$$

which is a contradiction. Hence, a stationary path with  $k > k^*$  cannot be an equilibrium.

Next consider the opposite case  $k < k^*$  in which  $\beta f'(k) > 1$ . Utility must not rise by reducing  $c_t, c_{t+1}, \dots$  and using the resulting savings to increase  $c_{t+T}$  in the future. We have

$$\phi \geq \max_T \left\{ \beta^T \frac{1-f'(k)^{T+1}}{1-f'(k)} \left[ \frac{1-\alpha}{\alpha} u'(c) + 1 - \beta\phi \right] + \beta^T - \frac{1-\alpha}{\alpha} \frac{1-\beta^T}{1-\beta} u'(c) \right\}.$$

Recall that we assumed  $\beta f'(k) > 1$ . Therefore, if  $\beta\phi < \frac{1-\alpha}{\alpha} u'(c) + 1$  (this condition holds if  $\beta\phi \leq \frac{1-\alpha}{\alpha} u'(c^*) + 1$ ), the right-hand side grows without bound as  $T \rightarrow \infty$  and the condition is violated. Hence, this cannot be an equilibrium either.

Instead, for high degrees of loss aversion,  $\beta\phi \geq \frac{1-\alpha}{\alpha} u'(c) + 1$ , the right-hand side is decreasing in  $T$  and stationary consumption paths below the steady state become possible.  $\square$

## References

- Alvarez-Cuadrado, F., Monteiro, G., Turnovsky, S., 2004. Habit formation, catching up with the Joneses and economic growth. *Journal of Economic Growth* 9, 47–80.
- Barberis, N.C., Huang, M., Santos, T., 2001. Prospect theory and asset prices. *Quarterly Journal of Economics* 116, 1–53.
- Barro, R.J., 1999. Ramsey meets Laibson in the neoclassical growth model. *Quarterly Journal of Economics* 114, 1125–1152.

- Benartzi, S., Thaler, R.H., 1995. Myopic loss aversion and the equity premium puzzle. *Quarterly Journal of Economics* 110, 73–92.
- Boyer, M., 1978. A habit forming optimal growth model. *International Economic Review* 19, 585–609.
- Camerer, C.F., 2004. Prospect theory in the wild: evidence from the field. In: Camerer, C.F., Loewenstein, G., Rabin, M. (Eds.), *Advances in Behavioral Economics*. Princeton University Press, Princeton, NJ, pp. 148–161.
- Cass, D., 1965. Optimum growth in an aggregative model of capital accumulation. *Review of Economic Studies* 32, 223–240.
- Carroll, C.D., Overland, J., Weil, D.N., 2000. Saving and growth with habit formation. *American Economic Review* 90, 341–355.
- De la Croix, D., Michel, P., 1999. Optimal growth when tastes are inherited. *Journal of Economic Dynamics and Control* 23, 519–537.
- Easterlin, R.A., 1974. Does economic growth improve the human lot? Some empirical evidence. In: David, P.A., Reder, M.W. (Eds.), *Nations and Households in Economic Growth: Essays in Honour of Moses Abramovitz*. Academic Press, New York, pp. 89–125.
- Easterlin, R.A., 1995. Will raising the incomes of all increase the happiness of all? *Journal of Economic Behavior and Organization* 27, 35–48.
- Easterlin, R.A., 2001. Income and happiness: towards a unified theory. *Economic Journal* 111, 465–484.
- Frank, R.H., 1997. The frame of reference as a public good. *Economic Journal* 107, 1832–1847.
- Fuhrer, J.C., 2000. Habit formation in consumption and its implications for monetary-policy models. *American Economic Review* 90, 367–390.
- Gervais, M., Klein, P., 2010. Measuring consumption smoothing in CEX data. *Journal of Monetary Economics*, forthcoming.
- Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decisions under risk. *Econometrica* 47, 263–291.
- Kahneman, D., Tversky, A., 2000. *Choices, Values and Frames*. Princeton University Press, Princeton, NJ.
- Koopmans, T.C., 1960. Stationary ordinal utility and impatience. *Econometrica* 28, 287–309.

- Koopmans, T.C., 1965. On the concept of optimal economic growth. In: *The Econometric Approach to Development Planning. Semaine d'étude sur le rôle de l'analyse économétrique dans la formulation de plans de développement*, Pontificiae Academiae Scientiarum Scripta Varia 28, 1 (also: Cowles Foundation Discussion Paper No. 163, 1963).
- Laibson, D., 1997. Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics* 112, 443–477.
- Layard, R., 2003. *Income and Happiness: Rethinking Economic Policy*. Lecture 2, LSE, London.
- Mausumi, D., 2003. Optimal growth with decreasing marginal impatience. *Journal of Economic Dynamics and Control* 27, 1881–1898.
- Ramsey, F.P., 1928. A mathematical theory of saving. *Economic Journal* 38, 543–559.
- Rosenblatt-Wisch, R., 2008. Loss aversion in aggregate macroeconomic time series. *European Economic Review* 52, 1140–1159.
- Ryder, H.E. Jr., Heal, G.M., 1973. Optimal growth with intertemporally dependent preferences. *Review of Economic Studies* 40, 1–31.
- Shi, S., Epstein, L.G., 1993. Habits and time preference. *International Economic Review* 34, 61–84.
- Surico, P., 2007. The Fed's monetary policy rule and U.S. inflation: the case of asymmetric preferences. *Journal of Economic Dynamics and Control* 31, 305–324.
- Stokey, N.L., Lucas, R.E. Jr., Prescott, E.C., 1989. *Recursive Methods in Economic Dynamics*. Harvard University Press, Cambridge, MA.
- Tversky, A., Kahneman, D., 1991. Loss aversion in riskless choice: a reference-dependent model. *Quarterly Journal of Economics* 106, 1039–1061.
- Tversky, A., Kahneman, D., 1992. Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty* 5, 297–323.
- Uzawa, H., 1968. Time preference, the consumption function, and optimum asset holdings. In: Wolfe, J.N. (Ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks*. Edinburgh University Press, Edinburgh.
- Yaniv, G., 1999. Tax compliance and advance tax payments: a prospect theory analysis. *National Tax Journal* 52, 753–764.